A HYDRODYNAMIC THEORY OF BILAYER MEMBRANE FORMATION

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ABSTRACT A simple formula for the expansion rate U of the bilayer membrane perimeter was derived by the solution of the hydrodynamic and the Laplace equations at small rates and small contact angles: $U = \theta\bar{\Delta}/6\mu \ln (h/e^{3/2}h_0)$, where θ is the equilibrium contact angle between the bilayer membrane and the Plateau-Gibbs border, $\bar{\Delta}$ the excess membrane tension, μ the bulk viscosity, h the thickness of the thick liquid film, and h_0 the thickness of the liquid core in the membrane. This formula coincides with the available experimental data, and explains the observed independence of the expansion rate on the membrane radius. The significance of the theory for the full description of the cell adhesion kinetics as well as the possible ways for its generalization are discussed.

The adhesion of a cell to an interface (particularly a cell surface) was considered mainly from its equilibrium aspects (Parsegian and Gingell, 1980). It may be expected that one of the important stages in the process of cell adhesion is the expansion of the three-phase contact line, similar to the process of attachment of a particle (bubble or drop) to another gas, liquid, or solid particle (Dimitrov, 1976; Ivanov, 1980). Although analyses in physiological conditions will ultimately prove necessary, it is initially easier and more instructive to use greatly simplified systems. In such systems it is possible to compare the theoretical predictions with well-characterized experimental data under strictly defined conditions. Because of the analogous geometry and the zero tangential surface velocity it seems that the formation of bilayer membranes is a process similar to the final stage of the cell adhesion process (at least from a hydrodynamic point of view). On the other hand, currently, black or bilayer lipid membranes are the focal point of a great variety of research activities, ranging from biological phenomena to solar energy conversion, and including measurements of electrical properties, permeabilities, photoelectric effects, etc. The problem of membrane formation kinetics has received little attention, however.

White (1972) and White et al. (1976) found the equations of mechanical equilibrium and the conditions for stability that must be satisfied in order for the membrane formation process to occur. Krugljakov et al. (1974) and recently Snyder et al. (1978) and Seccia et al. (1979) measured the rate of expansion of bilayer membranes at different conditions. Analogous measurements were performed previously with foam black films by Kolarov et al. (1968). The main conclusions were that in most cases the rate of expansion does not depend on the time t, and that it is proportional to the driving forces: the excess membrane tension $\bar{\Delta} = \Delta_0 - \Delta = 2\sigma(1 - \cos\theta)$, where Δ_0 is the tension of the thick film, Δ the membrane tension, σ the bulk surface tension, and θ the contact angle between the membrane and the surrounding Plateau-Gibbs border.

The first theoretical investigation of the expansion of black foam films, carried out by

Traykov et al. (1970), showed the proportionality between the driving force and the expansion rate U, but did not predict the independence of U on the t. The reason for the discrepancy between the theory and the experiment lay in the very simplified geometrical model of the interface shape, in which the black spot was considered as a piston, expanding axisymmetrically in the thick liquid film. In a latter paper Ivanov et al. (1978) described theoretically the secondary expansion (after covering the whole thick film by the spots) of a black film. They obtained a more realistic surface shape by solving simultaneously the hydrodynamic and the Laplace equations. Their results are in rather good agreement with the experimental data. There has been no theoretical work explaining in a simple way the experimental results about the bilayer membrane expansion up to now.

The approximate theory of the membrane formation kinetics presented in this paper in some aspects resembles the theory of dynamic contact angles (Hansen and Toong, 1971), and uses the ideas of the works of Traykov et al. (1970) and Ivanov et al. (1978). It predicts the linear dependence of the membrane radius R on the time t (with a constant expansion rate U) and coincides with the available experimental data. The basic assumptions we use are the following: (*a*) the membrane contains a liquid core of constant thickness $2h_0$. (*b*) During the expansion, the contact angle at the membrane perimeter is constant and equal to the equilibrium angle θ. (*c*) The radial component of the velocity goes to zero at the membrane perimeter. (*d*) The motion of the liquid is caused by the excess membrane tension $\bar{\Delta}$. (*e*) The thickness of the thick film $2h$ is constant and the hydrodynamic pressure p in it is equal to zero. In Fig. 1 the geometry of the assumed model in a cylindrical coordinate system is shown. Since, for a steady process, the local thickness h depends on t only through the instant value R (t) of the film radius, the velocity component v_z at $z = H$ is given by (Dimitrov and Ivanov,

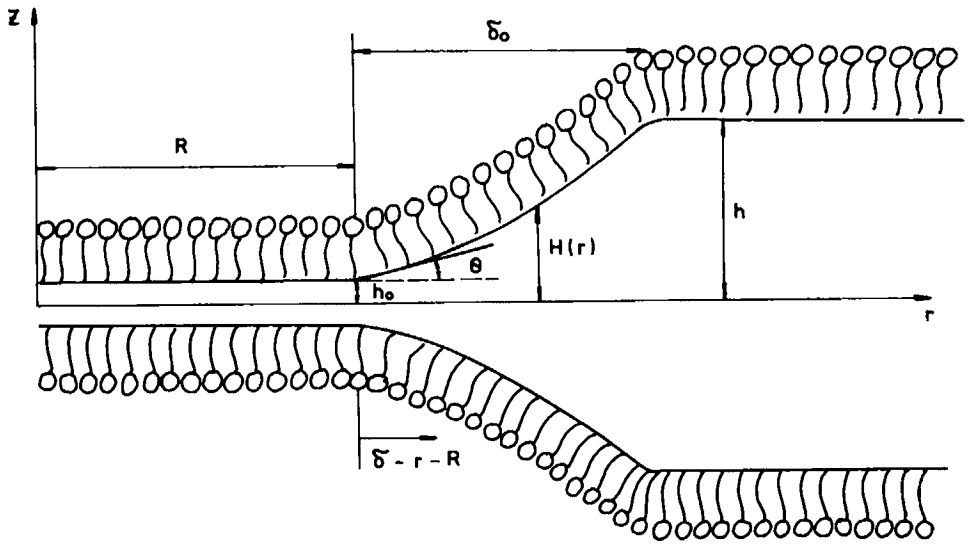

FIGURE 1 Sketch of the bilayer membrane and the thick liquid film in the vicinity of the perimeter of the membrane.

1978):

$$v_z = \partial H/\partial t + v_r(\partial H/\partial r) = U(\partial H/\partial R) + v_r(\partial H/\partial r), \tag{1}$$

where $U = \mathrm{d}R/\mathrm{d}t$ is the membrane expansion rate, v_r the radial velocity component and r the radial coordinate. Then the governing equation of the process will be (for details of the derivation see the works of Dimitrov [1976] and Dimitrov and Ivanov [1978]):

$$(\epsilon/H^3)\int_0^{\delta}(\partial H/\partial R)\mathrm{d}\delta = -(\partial p/\partial\delta)/\sigma = \partial^3 H/\partial\delta^3, \epsilon = 3n\mu U/\sigma, \tag{2}$$

where μ is dynamic viscosity, $\delta = r - R$, $n = 4$ for expansion of a membrane with tangentially immobile surface on a substratum, and $n = 1$ for expansion of a bilayer membrane or of a membrane on liquid-gas interface in the absence of surfactant. Eq. 2 was derived according to the lubrication theory equations and the Laplace equation in its differential form at small slopes $\partial H/\partial\delta$ ($\partial H/\partial\delta \ll 1$), i.e., the contact angle θ was assumed to be much smaller than unity. Another approximation employed when deriving Eq. 2 is $\delta \ll R$.

Since in most cases $U \gtrsim 10^{-1}$ cm/s the parameter ϵ does not usually exceed 10^{-3}. This allows Eq. 2 to be solved by means of an iteration procedure. The zero approximation $H^{(0)}$ (at $\epsilon = 0$) gives the equilibrium shape of an idealized system

$$H^{(0)} = h_0 + \theta\delta + \delta^2/R_c, \tag{3}$$

where R_c is the radius of curvature of the equilibrated meniscus surrounding the thick film (in the case when $R_c \gg R$). In this system the contribution of the long-range intermolecular forces (the disjoining pressure) is taken into account by introducing the equilibrium contact angle θ (Ivanov and Toshev, 1975; de Feijter et al., 1978). At small thickness $h = H(\delta_0)$ and at rather great contact angle θ we can assume $h \ll R_c\theta^2$, i.e.,

$$H^{(0)} = h_0 + \theta\delta. \tag{4}$$

We calculate the rate of expansion U from the integral balance of forces acting on the meniscus surface

$$\bar{\Delta} = 2\theta\int_0^{\delta_0} p\mathrm{d}\delta = -2\theta\int_0^{\delta_0}\delta(\partial p/\partial\delta)\,\mathrm{d}\delta, \tag{5}$$

where the assumption $p(\delta_0) = 0$ is used. The substitution of Eq. 4 in the left side of Eq. 2 and of the obtained expression in Eq. 5 yields:

$$\bar{\Delta}\theta/2\epsilon\sigma = ln(h/h_0) - 3/2 + 2h_0/h - h_0^2/2h^2. \tag{6}$$

At very small thicknesses h_0 we can assume $h_0 \ll h$. Then Eq. 6 gives:

$$U = \bar{\Delta}\theta/6n\mu ln(h/e^{3/2}h_0). \tag{7}$$

It can be seen from Eq. 7 that the expansion rate U is independent of radius R and time t, which is the basic experimental foundation. Eq. 7 coincides with the observation of Krugljakov and Rovin (1978) that the expansion rate is greatest when contact angle θ and excess tension $\bar{\Delta}$ are great, and viscosity μ is small. The quantitative comparison with the

available experimental data needs full information about the values of the contact angle θ, the excess tension $\bar{\Delta}$, the bulk viscosity μ, and the thicknesses h_0 and h of the liquid core in the bilayer membrane and of the thick film, respectively. Since in most experimental investigations these parameters were not measured simultaneously, we compare our theory with the data of Kolarov et al. (1968) and Krugljakov and Rovin (1978), where only the thickness h_0 was not determined. Fortunately, the dependence of the expansion rate U on the thickness h_0 is very weak (logarithmic) and the choice of the value of h_0 does not appreciably influence the calculated value of the rate U.

Kolarov et al. (1968) measured the rate of expansion of bilayer membranes ($n = 1$) obtained from a water solution of Na lauryl sulphate equal to 6×10^{-4} cm/s, when $\theta = 0.77°$, $\sigma = 48.4$ dyn /cm, $h = 3 \times 10^{-6}$ cm. The viscosity μ and the excess tension $\bar{\Delta}$ are equal to 10^{-2} p and 8×10^{-3} dyn/cm, respectively. Assuming a value of the order of 10^{-8} cm for h_0, from Eq. 7 we obtain $U = 4 \times 10^{-4}$ cm/s, which is of the order of the experimentally measured value.

Krugljakov and Rovin (1978) found that the rate of expansion of xylan-0 membranes in an 0-xylol/water system is equal to 5×10^{-4} cm/s. In this system the contact angle and the interface tension are equal to 3^0 and 2 dyn/cm, respectively. Assuming values of the order of 10^{-2} p and 10^{-8} cm for the bulk viscosity μ and the thickness h_0, respectively, from Eq. 7 we obtain $U = 8 \times 10^{-4}$ cm/s, which coincides with the measured value.

Unfortunately, there are no experimental data on the adhesion kinetics, especially on the three-phase contact line expansion. It seems that in this case the expansion rates U are very small (of order of 10^{-5} cm/s, obtained by assuming $\theta \sim 1^0$, $\mu \sim 10^{-2}$ p, $\sigma \sim 1$ dyn/cm, $h \sim 10^{-5}$ cm, and $h_0 \sim 10^{-8}$ cm). Hence, the duration of this stage will be of the order of $L/U \sim 100$ s (L is assumed to be the length of the cell of the order of 10^{-3} cm). This value is rather great, which shows the possible importance of the expansion stage in the process of cell adhesion.

From a theoretical point of view the following reasons for the possible discrepancy between theory and experiment in some investigations must be pointed out: (*a*) the hydrodynamic pressure in the thick film is not equal to zero. It seems that the dissipated energy in this region can not be neglected, especially in the case of great contact angle θ. Since the characteristic radius of this region is of the order of the membrane radius (at least in the initial stages of the membrane expansion), the hydrodynamic resistence will depend on the membrane radius, and the rate of expansion will be time-dependent in this case. It is interesting to note that when the black spot appears in the vicinity of the Plateau-Gibbs border, it expands to its boundary more rapidly. This may be explained by the lower hydrodynamic resistence in this region than the one on the other side of the spot. (*b*) Usually, the thickness of the thick film changes during the expansion, under the action of the disjoining and the capillary pressures. This change may be caused also by the action of the hydrodynamic forces, owing to the membrane perimeter expansion. (*c*) At zero thickness of the liquid core h_0 and at rather great expansion rates, the contact angle can differ from this at equilibrium (Blake and Haynes, 1969). In this case the theory must be slightly modified, but its basic features remains the same. (*d*) The influence of the line tension must be taken into account when calculating the excess membrane tension for very small black spots. (*e*) The liquid transport through the interfaces will increase the expansion rate, especially when the driving force is great. This could be important when considering the cell adhesion kinetics because of the rather great membrane permeability for

water and solutes. In experimental conditions, where the two liquid phases are usually not miscible, this effect probably does not play any significant role, exluding the possible influence of interfacial instability processes. (*f*) The layer, separating the two phases may have different mechanical properties from those of a single interface, characterized by its surface tension. This is important when considering the cell adhesion kinetics, where the specific mechanical properties of the cell membrane must be taken into account (Evans and Skalak, 1980).

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